DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

A PLANNED COURSE FOR:

ADVANCED PLACEMENT CALCULUS (BC)

Grade: 12

Date of Board Approval: _____2015_____________

DELAWARE VALLEY SCHOOL DISTRICT

PLANNED INSTRUCTION

Title of Planned Instruction: Advanced Placement Calculus (BC)

Subject Area: Mathematics Grade Level: 12

Course Description:

This course is intended to teach students the fundamental theories and applications of single variable calculus of the real number system. While the material described herein represents the set of expected standards for students in this course, a major goal for Advanced Placement Calculus is to provide students the necessary skills for satisfactory completion of the College Board's Advanced Placement Calculus (BC) Test in May of the school year. In the mathematics program at Delaware Valley, Advanced Placement Calculus follows the course in Honors Pre Calculus Mathematics.

Time/Credit for the Course: 46 minutes per day for a full year/ 1 credit

Curriculum Writing Committee: Gary Dennis

AP Calculus (BC) Course Syllabus

In order to meet Delaware Valley School District requirements and Advanced Placement standards the following sections of Calculus by Ross L. Finney, Franklin D. Demana, Bert K. Waits, and Daniel Kennedy must be covered before the Advanced Placement exam is administered in May:

Chapter 1 Prerequisites for Calculus

1.1 Lines
1.2 Functions and Graphs
1.3 Exponential Functions ** Chapter 1 is a review**
1.4 Parametric Equations
1.5 Functions and Logarithms
1.6 Trigonometric Functions

Chapter 2 Limits and Continuity

2.1 Rates of Change and Limits
2.2 Limits Involving Infinity
2.3 Continuity
2.4 Rates of Change and Tangent Lines

Chapter 3 Derivatives

3.1 Derivative of a Function
3.2 Differentiability
3.3 Rules of Differentiation
3.4 Velocity and Other Rates of Change
3.5 Derivatives of Trigonometric Functions
3.6 Chain Rule
3.7 Implicit Differentiation
3.8 Derivatives of Inverse Trigonometric Functions
3.9 Derivatives of Exponential and Logarithmic Functions

Chapter 4 Applications of Derivatives

4.1 Extreme Values of Functions
4.2 Mean Value Theorem
4.3 Connecting f' and f'' with the graph of f
4.4 Modeling and Optimization
4.5 Linearization and Newton's Method
4.6 Related Rates

Chapter 5 The Definite Integral

5.1 Estimating with Finite Sums
5.2 Definite Integrals
5.3 Definite Integrals and Antiderivatives
5.4 Fundamental Theorem of Calculus
5.5 Trapezoidal Rule

Chapter 6 Differential Equations and Mathematical Modeling

6.1 Slope Fields
6.2 Antidifferentiation by Substitution
6.3 Antidifferentiation by Parts
6.4 Exponential Growth and Decay
6.5 Logistic Growth

Chapter 7 Applications of Definite Integrals

7.1 Integral as Net Change
7.2 Areas in the Plane
7.3 Volumes
7.4 Lengths of Curves
7.5 Applications from Science and Statistics

Chapter 8 Improper Integrals

8.1 Sequences
8.2 L'Hopital's Rule
8.3 Relative Rates of Growth
8.4 Improper Integrals

Chapter 9 Infinite Series

9.1 Power Series
9.2 Taylor Series
9.3 Taylor's Theorem
9.4 Radius of Convergence
9.5 Testing Convergence at Endpoints

Chapter 10 Parametric, Vector, and Polar Functions

10.1 Parametric Functions
10.2 Vectors in the Plane
10.3 Polar Functions

Additional resource books and problems include:

- Calculus by Howard Anton
- Calculus from Graphical, Numerical, and Symbolic Points of View by Arnold Ostebee and Paul Zorn
- Preparing for the AP Calculus Examination by George W. Best and J. Richard Lux
- Multiple Choice and Free-Response Questions in Preparation for the AP Calculus Exam by David Lederman
- Master the AP Calculus Test by W. Michael Kelle
- Previous Advanced Placement exam problems, some of which can be found at the CollegeBoard website (www.apcentral.collegeboard.com).

Curriculum Map

1. **Marking Period One – 45 days**
    - Review of prerequisite topics from Pre-Calculus such as functions and their properties, equations of lines, analysis of quadratic, polynomial, exponential, logarithmic, and trigonometric functions. Evaluation and interpretation of limits, analysis of continuous functions, introduction to derivatives, methods of differentiation, and uses of derivatives.

    Marking Period One - Goals:
    Understanding of:
    - Review of prerequisite topics – equations of lines, analysis of quadratic, polynomial, exponential, logarithmic, and trigonometric functions (Minimal class time should be spent reviewing these topics.)
    - Intuitive approach to evaluating and determining limits
    - Analytical approaches to evaluating limits
    - Limits involving infinity
    - Relative rates of growth of functions that go toward infinity
    - Continuity of functions at a point
    - The Intermediate Value Theorem and its consequences
    - Average rate of change compared to instantaneous rate of change
    - Equations of tangent lines and normal lines
    - The definition of a derivative
    - The relationship between the graph of a function and the graph of its derivative
    - Basic methods and rules of differentiation
    - Higher order derivatives
    - Implicit Differentiation
    - Logarithmic Differentiation
    - Using derivatives to solve word problems
    - L'Hopital's Rule as a method for evaluating limits that are in an indeterminate form
    - Derivatives as the instantaneous rate of change of a quantity
    - Related rates problems

2. **Marking Period Two – 45 days**
    - Applications of derivatives, including related rates problems, function analysis, optimization, the Mean Value Theorem, and linear motion
    - Evaluation methods of antiderivatives, estimation of area under a curve using Riemann Sums, properties of integrals, and the Fundamental Theorem of Calculus

    Marking Period Two - Goals:
    Understanding of:
    - Analysis of functions and their graphs using the ideas of increasing/decreasing, relative extrema, concavity, and inflection points
    - The Mean Value Theorem and its application to real world problems
    - Global behavior of functions, including the analysis of absolute extrema
    - Application of derivatives to applied optimization problems
    - Analysis of linear motion using derivatives to describe velocity and acceleration and their applications to left/right movement and speeding up/slowing down
    - Methods of finding antiderivatives, including basic rules, guess-check-and-adjust method, and substitution
    - Estimation of the area under a curve using Riemann Sums (left endpoint, right endpoint, midpoint, and trapezoid methods)
    - Properties of definite integrals
    - Evaluation of definite integrals using geometric formulas
    - The Fundamental Theorem of Calculus (First Version) and its use for evaluating definite integrals

3. **Marking Period Three – 45 days**
    - Applications of integrals, including area between two curves, volumes of solids, linear motion, and differential equations
    - Infinite series

    Marking Period Three - Goals
    Understanding of:
    - The Fundamental Theorem of Calculus (Second Version) and its use for analyzing functions defined as integrals
    - Average value of a function
    - The Mean Value Theorem for integrals and its applications
    - The area between two curves, both in terms of x and y
    - Volumes of solids of revolution
    - Volumes of solids with known cross sections
    - Definite integral of a rate of change of a quantity as a method for evaluating the total change in the quantity

- Integration as a method for analyzing linear motion problems
- Identifying, interpreting, and solving differential equations
- Application of differential equations to real world problems, including exponential growth and decay problems
- Visualization of solutions to differential equations using slope fields
- Convergence and divergence of infinite sequences and series of constants
- Geometric series and their convergence/divergence
- Telescoping series
- Convergence tests for infinite series
- Absolute convergence for infinite series
- The Ratio Test for convergence of a power series
- Intervals of convergence for power series
- Maclaurin and Taylor Series
- Methods of manipulating Taylor Series to create new ones
- The Lagrange error bound for Taylor polynomials

4. Marking Period Four – 45 days

- Parametric equations
- Polar equations
- Reteaching, review and preparation for the Advanced Placement Exam
- Extension and enrichment topics after the Advanced Placement Exam

Marking Period Four - Goals:
Understanding of:

- Planar curves that are presented in parametric form
- Analysis of parametric equations
- Length of a curve given in parametric form
- Analysis of polar equations
- Area of a region formed by polar equations
- Using the graphing calculator to plot and analyze parametric and polar curves

Marking Period	Topics to be Covered
First Marking Period (Topics in Chapter 1 should be assigned to students as a review. Minimal class time should be spent on this chapter.)	**PRE-CALCULUS REVIEW** **Chapter 1** 1.1 Increments; Slope of a Line; Parallel and Perpendicular Lines; Equations of Lines; Applications 1.2 Functions; Domains and Ranges; Viewing and Interpreting Graphs; Even Functions and Odd Functions-Symmetry; Functions Defined in Pieces; Absolute Value Function; Composite Functions 1.3 Exponential Growth; Exponential Decay; Applications; The Number e 1.5 One-to-one Functions; Inverses; Finding Inverses; Logarithmic Functions; Properties of Logarithms; Applications 1.6 Radian Measure; Graphs of Trigonometric Functions; Periodicity; Even and Odd Trigonometric Functions; Transformations of Trigonometric Graphs; Inverse Trigonometric Functions **LIMITS AND CONTINUITY** **Chapter 2** 2.1 Average and Instantaneous Speed; Definition of Limit; Properties of Limits; One-sided and Two-sided Limits; Sandwich Theorem 2.2 Finite Limits as x approaches infinity; Sandwich Theorem Revisited; Infinite Limits as x approaches a; End Behavior Models; “Seeing” Limits as x approaches infinity **Chapter 8** 8.3 Comparing Rates of Growth **Chapter 2** 2.3 Continuity at a Point; Continuous Functions; Algebraic Combinations; Composites; Intermediate Value Theorem for Continuous Functions

	DERIVATIVES **Chapter 2** 2.4 Average Rates of Change; Tangent to a Curve; Slope of a Curve; Normal to a Curve; Speed Revisited **Chapter 3** 3.1 Definition of a Derivative; Notation; Relationship Between the Graphs of f and f'; Graphing the Derivative from Data; One-sided Derivatives 3.2 How f'(a) Might Fail to Exist; Differentiability Implies Local Linearity; Derivatives on a Calculator; Differentiability Implies Continuity; Intermediate Value Theorem for Derivatives 3.3 Positive Integer Powers, Multiples, Sums, and Differences; Products and Quotients; Negative Integer Powers of x; Second and Higher Order Derivatives 3.4 Instantaneous Rates of Change; Motion along a Line; Sensitivity to Change; Derivatives in Economics 3.5 Derivative of the Sine Function; Derivative of the Cosine Function; Simple Harmonic Motion; Jerk; Derivatives of Other Basic Trigonometric Functions 3.6 Derivative of a Composite Function; "Outside-Inside" Rule; Repeated Use of the Chain Rule; Power Chain Rule 3.7 Implicitly Defined Functions; Lenses, Tangents, and Normal Lines; Derivatives of Higher Order; Rational Powers of Differentiable Functions 3.8 Derivatives of Inverse Functions; Derivative of the Arcsine; Derivative of the Arctangent; Derivative of the Arcsecant; Derivatives of the Other Three 3.9 Derivative of e^x; Derivative of a^x; Derivative of $\ln x$; Derivative of $\log_a x$; Power Rule for Arbitrary Real Powers 8.2 L'Hopital's Rule **APPLICATIONS OF DERIVATIVES** **Chapter 4** 4.7 Related Rate Equations; Solution Strategy; Simulating Related Motion

Second Marking Period	**APPLICATIONS OF DERIVATIVES** **Chapter 4** 6.2 Absolute Extreme Values; Local Extreme Values; Finding Extreme Values 6.3 Mean Value Theorem; Physical Interpretation; Increasing and Decreasing Functions; Other Consequences 6.4 First Derivative Test for Local Extrema; Concavity; Points of Inflection; Second Derivative Test for Local Extrema; Learning about Functions from Derivatives 6.5 Examples from Mathematics; Examples from Business and Industry; Examples from Economics; Modeling Discrete Phenomena with Differentiable Functions 6.6 Linear Approximation; Differentials; Estimating Change with Differentials; Absolute, Relative, and Percentage Change; Sensitivity to Change **INTEGRALS** **Chapter 6** 6.2 Indefinite Integrals; Leibniz Notation and Antiderivatives; Substitution in Indefinite Integrals 6.3 Antidifferentiation by Parts 6.5 Partial Fractions **Chapter 5** 5.2 Riemann Sums; Terminology and Notation of Integration 5.5 Trapezoidal Approximations 5.2 Definite Integral and Area; Constant Functions; Integrals on a Calculator; Discontinuous Integrable Functions 5.3 Properties of Definite Integrals; Average Value of a Function; Mean Value Theorem for Definite Integrals; Connecting Differential and Integral Calculus 5.4 Fundamental Theorem, Part I; Graphing the function $\int_a^x f(t)dt$; Fundamental Theorem, Part II; Area connection; Analyzing Antiderivatives Graphically **Chapter 6** 6.2 Substitution in Definite Integrals **Chapter 8** 8.4 Improper Integrals

Third Marking Period	**APPLICATIONS OF INTEGRALS** **Chapter 7** 7.1 Linear Motion Revisited; General Strategy; Consumption Over Time; Net Change from Data 7.2 Area Between Curves; Area Enclosed by Intersecting Curves; Boundaries with Changing Functions; Integrating with Respect to y; Saving Time with Geometry Formulas 7.3 Volume As an Integral; Square Cross Sections Circular Cross Sections; Cylindrical Shells; Other Cross Sections 7.4 A Sine Wave; Length of a Smooth Curve; Vertical Tangents, Corners, and Cusps **DIFFERENTIAL EQUATIONS AND APPLICATIONS** **Chapter 6** 6.1 Differential Equations; Slope Fields; Euler's Method 6.4 Separable Differential Equations; Law of Exponential Change; Continuously Compounded Interest; Radioactivity; Modeling Growth with Other Bases; Newton's Law of Cooling 6.5 How Populations Grow; The Logistic Differential Equation; Logistic Growth Models **SEQUENCES** 8.1 Defining a Sequence; Arithmetic and Geometric Sequences; Graphing a Sequence; Limit of a Sequence **SERIES** 9.1 Geometric Series 9.4 Convergence; Comparing Nonnegative Series; Ratio Test 9.5 Integral Test; Harmonic Series and *p*-series; Comparison Tests; Alternating Series; Absolute and Conditional Convergence 9.1 Representing Functions by Series 9.4 Endpoint Convergence 9.1 Differentiation and Integration; Identifying a Series 9.2 Constructing a Series; Series for sin *x* and cos *x*; Beauty Bare; Maclaurin and Taylor Series; Combining Taylor Series; Table of Maclaurin Series

	9.3 Taylor Polynomials; The Remainder; Remainder Estimation Theorem; Euler's Formula
Fourth Marking Period	**Parametric, Vector, and Polar Functions** 10.1 Parametric Curves in the Plane; Slope and Concavity; Arc Length 10.2 Two-dimensional Vectors; Vector Operations; Modeling Planar Motion; Velocity, Acceleration, and Speed; Displacement and Distance Traveled 10.4 Polar Coordinates; Polar Curves; Slopes of Polar Curves; Areas Enclosed by Polar Curves; A Small Polar Gallery

UNIT 1: Limits and Continuity

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 2: Differentiation

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 3: Applications of Differentiation

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 4: Integration and the Fundamental Theorem of Calculus

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 5: Applications of Integration

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 6: Series

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

UNIT 7: Parametric and Polar Equations

Refer to the Curriculum Framework for Advanced Placement Calculus provided by the College Board for Big Ideas, Essential Questions, Concepts, and Competencies:

https://secure-media.collegeboard.org/digitalServices/pdf/ap/ap-calculus-curriculum-framework.pdf

Curriculum Plan

Unit 1: Limits and Continuity

Time Range in Days: 15 days

Overview: This unit emphasizes the idea of a limit, which is the underlying concept to all of calculus. The focus is on methods of evaluating limits, interpretations of limits, and the application of limits to continuous functions. Since the condition of continuity for functions is essential for many later concepts in calculus, the implications and applications of continuous functions will also be emphasized.

Focus Questions:
- What is meant by a limit?
- How can limits be evaluated?
- How can limits be applied to real world situations?
- What is meant by a continuous function?
- What are the different types of discontinuities?
- What implications and applications do continuous functions have?

Goals: Students will be able to
- Evaluate and interpret limits numerically, analytically, and graphically
- Explain the idea of continuous functions
- Use limits to determine points of discontinuity on a graph
- Apply theorems that rely on the idea of continuity

Objectives: Students will be able to
- Find the average rate of change of a function on a closed interval. (DOK Level Two)
- Intuitively understand the limiting process. (DOK Level Two)
- Calculate limits (including one-sided limits) using algebra and properties of limits. (DOK Level Two)
- Estimate limits from graphs or tables of data. (DOK Level Two)
- Connect geometric and analytic information both to predict and to explain the observed local and global behavior of a function. (DOK Level Three)
- Apply the Squeeze (Sandwich or Pinch) Theorem. (DOK Level Four)
- Using a graphing calculator, plot the graph of a function within an arbitrary window. (DOK Level Two)
- Using a graphing calculator, find the zeros of a function (solve equations numerically). (DOK Level Two)
- Understand asymptotes in terms of graphical behavior. (DOK Level Two)
- Describe asymptotic behavior in terms of limits involving infinity. (DOK Level Three)

- Compare relative magnitudes of functions and their rates of change (without using L'Hopital's Rule). (DOK Level Three)
- Intuitively understand the concept of continuity. (DOK Level Two)
- Understand continuity in terms of limits. (DOK Level Three)
- Distinguish between removable discontinuities and non-removable discontinuities. (DOK Level Two)
- Apply properties of continuous functions. (DOK Level Four)
- Understand the geometry of graphs of continuous functions. (DOK Level Three)
- Apply the Intermediate Value Theorem and it consequences. (DOK Level Four)

Core Activities and Corresponding Instructional Methods:

- Evaluate and interpret limits

    - Present graphs of non-linear functions to compare and contrast the ideas of average and instantaneous rates of change

    - Use a graphing calculator to create scatter plots that aid in explaining rates of change and introduce the intuitive idea of a limit

    - Provide graphs of functions that help to demonstrate the intuitive idea of a limit, both one-sided and two-sided, as well as limits as $x \rightarrow \propto$

    - Student group investigation and discussion of the evaluation and explanation of limits, both one-sided and two-sided, as well as limits as $x \rightarrow \propto$

    - Provide graphs of functions that demonstrate how a limit might not exist

    - Demonstrate graphically the Sandwich Theorem and how it applies to certain limits such as $\lim\limits_{x \rightarrow 0} x^2 \sin\left(\frac{1}{x}\right)$ and/or $\lim\limits_{x \rightarrow 0} \left(\frac{\sin x}{x}\right)$

    - Explain how to evaluate limits analytically, and how these solutions are visually consistent with their graphs

    - Guided practice on how to evaluate limits graphically and analytically

- Establish the concept of continuity and analyze its consequences

    - Provide examples of discontinuous functions to demonstrate visually what is meant by continuous and to explain different types of discontinuities

    - Define continuity at a point using limits

- Demonstrate graphically the Intermediate Value Theorem and its implications, emphasizing the continuity condition

- Guided practice on how to identify discontinuities of a function analytically and interpret them graphically

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework
- Quizzes

Summative: Common Assessment for Unit 1

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 59-86, 453-458)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, p. 59-86, 453-458
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

Unit 2: Differentiation

Time Range in Days: 25 days

Overview: This unit develops the idea of differentiation by first comparing the average rate of change of a function to the instantaneous rate of change. The concept of a derivative is presented using limits, and methods of differentiation will be formulated and proven.

Focus Questions:
- What is the difference between average rate of change and instantaneous rate of change?
- How can the instantaneous rate of change of a function be evaluated?
- What is a derivative?
- How can derivatives be determined?
- How might a derivative not exist?
- What implications and applications do derivatives have?

Goals: Students will be able to
- Find the average rate of change of a function
- Explain the idea of instantaneous rate of change of a function
- Estimate the instantaneous rate of change of a function at a point
- Explain derivatives as the limit of a secant line as one point moves closer to another point
- Evaluate derivatives using appropriate techniques
- Use derivatives to solve word problems

Objectives: Students will be able to
- Interpret the slope of a secant line as the average rate of change of a function. (DOK Level Two)
- Interpret the slope of a tangent line as the instantaneous rate of change of a function. (DOK Level Two)
- Explain the difference between average rate of change and instantaneous rate of change of a function (DOK Level Three)
- Determine the slope of the tangent line to a curve at a point and the equation of this line. (DOK Level Three)
- Determine the equation of the normal line to a curve at a point. (DOK Level Two)
- Utilize the equation of a tangent line to extrapolate approximate values of a function. (DOK Level Three)
- Calculate instantaneous rate of change by using the limit of average rate of change. (DOK Level Three)

- Approximate the rate of change of a function from graphs and table of values. (DOK Level Two)
- Present derivatives graphically, numerically, and analytically. (DOK Level Three)
- Interpret the derivative as an instantaneous rate of change. (DOK Level Two)
- Apply the definition of a derivative as the limit of the difference quotient. (DOK Level Three)
- Determine and justify whether a curve has points at which there is a vertical tangent line or no tangent line. (DOK Level Three)
- Recognize and explain the relationship between differentiability and continuity. (DOK Level Three)
- Using a graphing calculator, numerically calculate the derivative of a function at a specified value. (DOK Level Two)
- Identify differentiable functions as also being continuous. (DOK Level One)
- Find derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions. (DOK Level Two)
- Find derivatives of functions using basic rules for sums, products, and quotients. (DOK Level Two)
- Find derivatives of functions using the chain rule and implicit differentiation. (DOK Level Three)
- Find derivatives of functions using logarithmic differentiation. (DOK Level Three)
- Find derivatives of inverse functions. (DOK Level Three)
- Evaluate limits using L'Hopital's Rule and explain when it is an appropriate method to use. (DOK Level Three)

Core Activities and Corresponding Instructional Methods:

- Evaluate and interpret rates of change
    - Present graphs of non-linear functions to compare and contrast the ideas of average and instantaneous rates of change
    - Student group investigation and discovery that real world situations that highlight the uses and differences between average and instantaneous rates of change
    - Explain how the instantaneous rate of change of a function can be determined using the average rate of change as an interval is made smaller, emphasizing the role of limits in this description
    - Explain that the instantaneous rate of change is the slope of the tangent line to a curve

- Guided practice on estimating average and instantaneous rates of change graphically, numerically, and analytically

- Guided practice on interpreting average and instantaneous rates of change in applied situations

- Define and explain differentiation (finding derivatives of functions)
  - Draw tangent lines at various points on the graph of a curve, and plot the slopes of these lines on a separate graph to introduce the idea of a derivative
  - Use the graph of a parabola and tangent lines at various points to demonstrate that the derivative of a quadratic function is a linear function
  - Present differentiation as a method for evaluating the instantaneous rate of change of a function
  - Interpret derivatives using graphs, tables of values, and analytic functions
  - Use a graphing calculator to determine the derivative of a function at a given point
  - Explain the role of continuity in determining whether a function is differentiable, using specific examples to demonstrate when a function is not differentiable
  - Provide graphs of functions to demonstrate why the derivative at a given point might not exist (for example, discontinuous functions and corners on graphs such as with $y = |x|$)
  - Use a graphing calculator to zoom in on a differentiable graph and its tangent line at a point to demonstrate local linearity

- Develop methods for differentiation (finding derivatives)
  - Present basic methods of differentiation: Power Rule, properties of derivatives (constants, sums, differences, constant factors), the Product Rule, the Quotient Rule, and the Chain Rule
  - Show graphically that the Power Rule is consistent with constant functions

- Demonstrate graphically that a vertical shift of a graph does not affect its derivative, indicating that the derivative of $f(x) + c$ is the same as the derivative of $f(x)$

- Present examples of functions for which the Product and/or Quotient Rules apply

- Independent practice with methods of differentiation and peer assessment of student progress

- Guided practice with methods of differentiation

- Demonstrate different notations for higher order derivatives

- Prove the formula for the derivative of sine analytically

- Prove the formula for the derivative of cosine graphically

- Prove the formulas for cosecant, secant, tangent, and/or cotangent using the derivative formulas for sine and cosine, the Quotient Rule, and trigonometric identities

- Demonstrate the derivatives of $y = e^x$ and $y = \ln x$ graphically

- Provide a variety of examples involving powers, radicals, ratios, trigonometric functions, exponential functions, and logarithmic functions for which the Chain Rule applies

- Present implicit equations (for example, circles and/or ellipses) and demonstrate how to find a derivative implicitly

- Present exponential functions that have the independent variable in both the base and the exponent and demonstrate how to find the derivative of such functions using logarithmic differentiation

- Group practice evaluating derivatives of all types

- Demonstrate how to find the derivative of an inverse function analytically and verify this method graphically by showing the tangent lines to the original function and its inverse

- Present examples that confirm the consistency of L'Hopital's Rule with previous methods of determining limits (for example, $\lim\limits_{x \to 2} \dfrac{x^2 - 4}{x^2 - 6x + 8}$)

- Present examples that indicate when L'Hopital's Rule should not be used (for example, $\lim_{x \to 3} \frac{5}{x-3}$)

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework
- Quizzes

Summative: Common Assessment for Unit 2

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 87-184)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, p. 87-184
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

<u>Unit 3:</u> Applications of Differentiation

<u>Time Range in Days:</u> 30 days

Overview: This unit applies the idea and methods of differentiation to a variety of theoretical and real world problems.

Focus Questions:

- How can derivatives be used to solve theoretical problems?
- How do derivatives apply to real world situations?
- What is a related rates problem and how can it be solved?
- What information do derivatives provide about the graph of a function?
- How can relative extrema of functions be determined using derivatives?
- How can calculus be used to explain the global behavior of a function's graph?
- What is the Extreme Value Theorem and why is it true?
- What is the Mean Value Theorem and why is it true?
- How can the Mean Value Theorem be applied to real world situations?
- How can derivatives help to explain the motion of a particle moving along a straight line?

Goals: Students will be able to

- Use derivatives to solve word problems
- Explain how derivatives can be used to describe real world situations
- Solve related rates problems
- Analyze graphs of functions using the first and second derivatives
- Determine and explain the local and global extreme values of a function
- Explain and apply the Extreme Value Theorem and the Mean Value Theorem
- Analyze linear motion problems

Objectives: Students will be able to

- Interpret the derivative as a rate of change in varied applied contexts. (DOK Level Three)
- Approximate the value of a function at a point using the tangent line to the function at a nearby point. (DOK Level Three)
- Approximate the change in function value using differentials. (DOK Level Three)
- Model rates of change problems, including solving related rates problems. (DOK Level Four)
- Distinguish between the characteristics of f and f'. (DOK Level Three)
- Understand the relationship between the increasing and decreasing behavior of f and the sign of f'. (DOK Level Three)

- Apply the First Derivative Test to determine relative extrema of a function. (DOK Level Three)
- Identify intervals of concavity of a function using the second derivative of a function. (DOK Level Three)
- Determine points of inflection of a function. (DOK Level Two)
- Apply the Second Derivative Test to determine relative extrema of a function. (DOK Level Three)
- Identify critical points of a function. (DOK Level Two)
- Identify absolute extrema of a function. (DOK Level Two)
- Apply the Extreme Value Theorem. (DOK Level Three)
- Determine intervals of increase/decrease of a function. (DOK Level Three)
- Apply the Mean Value Theorem and its geometric consequences. (DOK Level Four)
- Identify and apply consequences of the Mean Value Theorem. (DOK Level Three)
- Optimize functions, finding both absolute (global) and relative (local) extrema. (DOK Level Three)
- Use derivatives to solve linear motion problems involving position, velocity, speed, and acceleration. (DOK Level Four)

Core Activities and Corresponding Instructional Methods:

- Use derivatives to solve word problems
    - Apply the concept of derivative to word problems involving instantaneous rates of change
    - Demonstrate how implicit differentiation with respect to time can be applied to related rates problems
    - Solve related rate problems given various initial conditions (in particular, expanding circles, squares, and/or rectangles, a sliding ladder, and/or a rising balloon and its angle of elevation)
    - Group investigation and practice with related rates problems
- Use derivatives to analyze functions and their graphs
    - Demonstrate graphically the difference between absolute and relative extrema
    - Graphically define intervals of increase/decrease of various functions
    - Apply the first derivative to various functions to determine relative extrema

  - Demonstrate graphically the idea of concavity and explain how the second derivative can be used to determine intervals of concave up/down

  - Present graphs of functions that contain inflection points and emphasize that they are points where there is a change in concavity

  - Guided and group practice of function analysis problems using derivatives and their applications

  - Solve optimization problems that can be solved by applying derivative techniques (for example, maximizing area of a rectangular field, minimizing the surface area of a solid, and/or optimizing profit/cost in economics problems)

  - Guide and group practice solving linear motion problems

- Use derivatives to analyze linear motion problems

  - Demonstrate the Mean Value Theorem graphically and in the context of rates of change (for example, average velocity over a time interval compared to the instantaneous velocity at certain moments)

  - Use derivatives to describe the motion of a particle along a line, emphasizing the concepts of velocity, speed, and acceleration

  - Present linear motion problems graphically, numerically, and analytically

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework
- Quizzes

Summative: Common Assessment for Unit 3

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 187-260)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, p. 187-260
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

Unit 4: Integrals and the Fundamental Theorem of Calculus

Time Range in Days: 20 days

Overview: This unit develops the idea of antidifferentiation (integration), methods of integration, and interpretations of integration.

Focus Questions:

- How can the process of differentiation be reversed?
- What are the methods for integrating a function?
- What is the difference between an indefinite and a definite integral?
- What are the properties of integrals?
- How can the area under a curve be estimated?
- When do given methods of estimating area under a curve provide an overestimate and when do they provide an underestimate to the actual area?
- How can the exact area under a curve be determined?
- What is the Fundamental Theorem of Calculus and how is it applied?
- How is the average value of a function evaluated?
- What is the Mean Value Theorem for Integrals and how can it be applied?

Goals: Students will be able to

- Find a function whose derivative is given, by logically reversing the methods of differentiation
- Integrate functions using basic properties, the guess-check-and adjust method, and the substitution method
- Integrate functions using partial fraction decomposition and integration by parts
- Estimate the area under a curve using the left endpoint, right endpoint, midpoint, and trapezoid methods
- Explain whether each method of estimating area under a curve provides an overestimate or underestimate
- Explain how the exact area under a curve can be determined using limits
- Explain the difference between indefinite and definite integrals
- Understand and use the Fundamental Theorem of Calculus to evaluate definite integrals
- Evaluate improper integrals
- Analyze functions defined as integrals by applying the Fundamental Theorem of Calculus
- Evaluate the average value of a function over a closed interval
- Explain and apply the Mean Value Theorem for Integrals

Objectives: Students will be able to

- Find antiderivatives of functions directly from derivatives of basic functions. (DOK Level Two)
- Find antiderivatives of functions by the guess-check-and adjust method. (DOK Level Three)
- Find antiderivatives of functions by the substitution of variables. (DOK Level Three)
- Find antiderivatives of functions by the partial fraction decomposition. (DOK Level Three)
- Find antiderivatives of functions by the integration by parts. (DOK Level Four)
- Use Sigma notation to represent sums of values. (DOK Level Two)
- Estimate area under a curve using left, right, and midpoint evaluation points. (DOK Level Three)
- Use trapezoidal sums to approximate area under functions represented algebraically, graphically, and by tables of values. (DOK Level Three)
- Approximate a definite integral by using an appropriate Riemann Sum. (DOK Level Three)
- Evaluate a definite integral by calculating the limit of a corresponding Riemann Sum over equal subdivisions. (DOK Level Three)
- Apply basic properties, such as additivity and linearity, to definite integrals. (DOK Level Two)
- Determine the sign of a definite integral based on the graph of the function and the given interval. (DOK Level Two)
- Use the Fundamental Theorem to evaluate definite integrals. (DOK Level Three)
- Evaluate a definite integral that requires substitution by using the Fundamental Theorem of Calculus and changing the limits of the definite integral. (DOK Level Three)
- Evaluate an improper integral. (DOK Level Three)
- Use the Fundamental Theorem to represent a particular antiderivative. (DOK Level Three)
- Analytically and graphically analyze functions defined as definite integrals. (DOK Level Four)
- Using a graphing calculator, numerically calculate the value of a definite integral. (DOK Level Three)
- Find and interpret the average value of a function on a closed interval. (DOK Level Three)
- Apply the Mean Value Theorem for Integrals. (DOK Level Three)

Core Activities and Corresponding Instructional Methods:

- Find the antiderivatives of given functions

    - Student group investigation and discovery of methods of antidifferentiation

- Demonstrate the Power Rule for Integrals by reversing the process of taking a derivative using the Power Rule

- Explain that integration techniques are the reverse process of derivative techniques, especially for polynomial, trigonometric, exponential, and logarithmic functions

- Demonstrate that the antiderivative of a more complicated function can be determined through guessing an answer, checking by taking the derivative, and adjusting for any constant factor

- Explain the integration technique of substitution as a method for reversing the Chain Rule process for derivatives and provide specific examples to demonstrate this connection

- Demonstrate the method of integration by partial fractions to find an antiderivative of a function with non-repeating, linear factors in the denominator

- Demonstrate the method of integration by parts to find an antiderivative of a function that is the result of a Product Rule for derivatives

- Guided practice with all integration methods

- Independent and group practice with all integration methods with peer assessment of student progress

* Estimate the area under a curve

  - Group investigation and discovery of methods for estimating area under a curve

  - Show that the area under a curve can be approximated using rectangles formed using left endpoints, right endpoints, or midpoints

  - Show that the area under a curve can be approximated using trapezoids

  - Independent and group practice with estimating the area under a curve using various methods with peer assessment of student progress

  - Demonstrate how each method of estimating area under a curve can lead to an overestimate or underestimate, depending on the increase/decrease/concavity of the graph

  - Use sigma notation to represent a Riemann Sum

- Explain that as more rectangles are used for a Riemann Sum, the approximation becomes closer to the actual area under the curve

- Evaluate the exact area under a curve using definite integrals
  - Explain and apply the Fundamental Theorem of Calculus as a method for evaluating definite integrals
  - Demonstrate properties of definite integrals both graphically and analytically
  - Guided practice of evaluating definite integrals using the Fundamental Theorem of Calculus
  - Independent and group practice with evaluating definite integrals using the Fundamental Theorem of Calculus
  - Prove the Fundamental Theorem of Calculus using the Mean Value Theorem and Riemann Sums
  - Evaluate definite integrals using a graphing calculator and verify the results using the Fundamental Theorem of Calculus
  - Analyze functions defined as integrals using the Fundamental Theorem of Calculus by investigating the increase/decrease/concavity/relative extrema/absolute extrema of the function
  - Explain the average value of a function as the height for which the resulting rectangle has the same area as the region under the curve
  - Demonstrate the Mean Value Theorem for integrals graphically as the intersection of the average value constant function and the original curve
  - Apply integration by substitution to definite integrals, including the change of limits of integration

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework

- Quizzes

Summative: Common Assessment for Unit 4

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 263-319, 331-347, 363-364, 459-466)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, (p. 263-319, 331-347, 363-364, 459-466)
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

Unit 5: Applications of Integrals

Time Range in Days: 20 days

Overview: This unit applies integration to a variety of situations, including area/volume, linear motion, and the net change in a quantity. Definite integrals will also be used to solve differential equations and to apply differential equations to real world situations.

Focus Questions:

- How does the limiting process of Riemann Sums lead to definite integrals, and how is this applied to real world situations?
- How can definite integrals be used to determine the area between two curves?
- What is the difference between viewing a function in terms of y instead of in terms of x, and how does this change how the area of a region is determined?
- How can definite integrals be used to determine the volume of a given solid of revolution?
- How does rotating a region around different axes change the solid of revolution formed, and how does this change how the volume can be determined?
- How can definite integrals be used to determine the volume of a solid with known cross-sections?
- How can definite integrals be used to determine the length of a plane curve?
- How can definite integrals be used to analyze linear motion models?
- Why does a definite integral of a rate of change of a quantity produce the total change in the quantity, and how can this be applied to real world situations?
- What are differential equations?
- How can integrals be used to solve differential equations?
- What are the real world applications of differential equations?
- What is meant by exponential growth or decay and how can they be analyzed using differential equations?

Goals: Students will be able to

- Apply the limiting process of a Riemann Sum to transition from an estimated value to the actual value in a variety of real world situations.
- Determine the area of a region bounded by two curves that are functions of y.
- Determine the area of a region bounded by two or more curves that require separate integrals in order to account for changes in the limits of integration and integrand functions.
- Calculate the volume of a solid of revolution, formed by rotating about various axes, using the slice method (solid circular disks or washers).
- Calculate the volume of a solid with known cross-sections.

- Apply the concept of using an integral of a rate of change to find accumulated change in a variety of contexts.
- Find the length of a curve in the plane using a definite integral.
- Use integrals to analyze linear motion problems.
- Calculate the displacement and distance traveled by a particle along a line.
- Solve separable differential equations and use them to model solutions.
- Solve exponential growth and decay problems.
- Solve logistic growth problems.
- Interpret differential equations geometrically by using slope fields.
- Sketch solution curves for differential equations using slope fields.
- Estimate function values using Euler's Method.

Objectives: Students will be able to
- Use Riemann Sums (right endpoint, left endpoint, midpoint, trapezoid methods) to estimate values in applied contexts. (DOK Level Two)
- Interpret the meaning of a Riemann Sums (right endpoint, left endpoint, midpoint, trapezoid methods) in applied contexts. (DOK Level Three)
- Explain whether and why a given Riemann Sums represents an overestimate or underestimate. (DOK Level Four)
- Determine the area of a region bounded by two curves that are functions of x. (DOK Level Three)
- Determine the area of a region bounded by two curves that are functions of y. (DOK Level Three)
- Explain why one integral may not be sufficient to evaluate the area of a region. (DOK Level Three)
- Evaluate the area of a region bounded by two or more curves that require separate integrals in order to account for changes in the limits of integration and integrand functions. (DOK Level Four)
- Calculate the volume of a solid of revolution, formed by rotating about various axes, using the slice method (solid circular disks). (DOK Level Three)
- Calculate the volume of a solid of revolution, formed by rotating about various axes, using the slice method (washers). (DOK Level Three)
- Calculate the volume of a solid with known cross-sections. (DOK Level Four)
- Determine the length of a curve in the plane given a function and an interval (DOK Level Three)
- Apply the concept of using a definite integral of a rate of change to find accumulated change in a variety of contexts. (DOK Level Three)
- Interpret and explain values of definite integrals of rates of change as the net change in a quantity. (DOK Level Four)
- Indicate the appropriate units of measure in applied problems. (DOK Level Two)
- Use integrals to determine the velocity and position functions from an acceleration function. (DOK Level Three)
- Explain the difference between displacement and distance traveled by a particle along a line. (DOK Level Two)

- Calculate the displacement and distance traveled by a particle along a line. (DOK Level Three)
- Use a graphing calculator to analyze linear motion problems. (DOK Level Three)
- Solve separable differential equations using the separation of variables technique. (DOK Level Four)
- Use differential equations to model applied problems. (DOK Level Three)
- Apply differential equations to exponential growth and decay problems. (DOK Level Three)
- Analyze exponential growth and decay problems. (DOK Level Four)
- Analyze logistic growth problems. (DOK Level Four)
- Create the slope field associated with a differential equation. (DOK Level Three)
- Sketch solution curves for differential equations using slope fields. (DOK Level Three)
- Interpret solutions to differential equations geometrically by using slope fields. (DOK Level Four)
- Estimate function values from a differential equation and an initial value using Euler's Method. (DOK Level Four)

Core Activities and Corresponding Instructional Methods:

- Find the area/volume of given regions/solids

    - Demonstrate the area between two curves graphically as the area under the upper curve minus the area under the lower curve and as a Riemann Sum in which the height of the rectangles is the difference between the upper and lower functions.

    - Provide examples of regions that are enclosed by multiple curves, in which one integral is not sufficient to determine the area due to a change in limits of integration and/or the integrand function(s) through the given interval.

    - Show that the integral of the absolute value of the difference of two functions will find the total area between the graphs of the two functions.

    - Guided and independent practice with finding the areas of given regions.

    - Present solids of revolution and explain the slice method of finding the volume as an extension of Riemann Sums.

    - Provide examples of regions that are not completely adjacent to the axis of revolution, and thus, whose cross-sections are washers instead of discs.

    - Present solids of revolution that are formed by rotating a given region about an axis other than the coordinate axes.

- Demonstrate solids with known cross-sections (squares, rectangles, equilateral triangles, right isosceles triangles, and/or semi-circles).

- Establish the general method of determining the volume of a solid with known cross-sections by estimating the volume of one sample slice of the solid, estimating the volume of the solid using a Riemann Sum, and converting this sum into a definite integral in order to find the exact volume.

- Guided practice in finding the area of given regions and volumes of solids of revolution and with known cross-sections.

- Group practice in finding the area of given regions and volumes of solids of revolution and with known cross-sections and peer assessment of student progress.

- Demonstrate how to find the length of a curve using Riemann Sums and derive its formula

- Guided practice with finding the length of a planar curve

- Use Riemann Sums and definite integrals to analyze to real world problems

  - Provide examples (graphically, numerically, and analytically) to demonstrate that when a rate of change of a quantity varies, Riemann Sums can used to estimate the total change in the quantity.

  - Demonstrate that the limiting process of a Riemann Sum can lead to an exact answer using definite integrals in applied problems.

  - Emphasize the appropriate units of measure in problems involving rates of change of a quantity and the determination of the change in the quantity.

  - Present, solve, and analyze applied problems in which two opposing rates of change are involved (for example, water flowing both into and out of a water tank or people both entering and exiting an amusement park)

  - Guided and group practice with solving and analyzing applied problems that involve varying rates of change.

- Solve and apply differential equations
  - Explain what a differential equation is and what it means to solve a differential equation
  - Direct instruction on solving differential equations using separation of variables and verify the solution using implicit differentiation.
  - Guided and group practice solving differential equations and verifying the solutions.
  - Emphasize the constant of integration and its importance to completely solving a differential equation when an initial condition is provided.
  - Present the population growth/radioactive decay model as the solution to the differential equation model associated with a quantity whose rate of growth/decay is proportional to the amount of the quantity present.
  - Simulate logistic growth using the random number generator in the graphing calculator to represent the spread of a disease.
  - Explain the characteristics of logistic growth and the associated graph.
  - Demonstrate slope fields as a method for visualizing solution curves to differential equations.
  - Guided and group practice with matching slope fields to their associated differential equation (and vice versa), as well as creating slope fields from a given differential equation and peer assessment of student progress.
  - Interpret the graphical behavior of solutions to differential equations using slope fields.
  - Demonstrate Euler's Method as a tangent line approximation that is repeated using small increments in order to simulate the curvature of a solution curve.

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework

- Quizzes

Summative: Common Assessment for Unit 5

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 321-330, 350-368, 378-416)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, (p. 321-330, 350-368, 378-416)
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

<u>Unit 6:</u> Series

<u>Time Range in Days:</u> 25 days

<u>Overview:</u> This unit investigates infinite sequences and the idea of convergence, and then focuses on series and methods for determining whether they converge. These ideas are extended to series involving variables, with an emphasis on the uses, applications, and limitations of such series, in particular Maclaurin and Taylor Series.

Focus Questions:

- What is a sequence?
- How can the convergence/divergence of a sequence be determined?
- What is a series?
- How can the convergence/divergence of a series be determined?
- How can a graphing calculator be used to investigate series?
- What is a geometric series?
- What are some applications of geometric series?
- How can the convergence/divergence of a geometric series be determined?
- How can a convergent geometric series be evaluated?
- What methods can be used to determine the convergence/divergence of series?
- What is the difference between conditional and absolute convergence?
- When does an alternating series converge?
- What is a power series?
- How can the radius and interval of convergence of a power series be determined?
- What is a Maclaurin Series?
- What is a Taylor Series?
- How can a Maclaurin Series or Taylor Series be determined?
- How can known Maclaurin Series be manipulated to create new series representations for functions?
- What is the Lagrange Error Bound Formula and how is it applied to Taylor Series?

Goals: Students will be able to

- Define a sequence.
- Determine the convergence/divergence of a sequence.
- Distinguish between a sequence and a series.
- Explain what is meant by the convergence of a series.
- Use a graphing calculator to investigate the convergence/divergence of a series.
- Define a geometric series.
- Determine the convergence/divergence of a geometric series.
- Evaluate a convergent geometric series.

- Apply geometric series.
- Evaluate a telescoping series.
- Recognize the harmonic series.
- Explain why the harmonic series diverges.
- Use the Comparison Test to check for convergence of a series.
- Use the Integral Test to check for convergence of a series.
- Check for convergence of a p-series.
- Use the Limit Comparison Test to check for convergence of a series.
- Use the Ratio Test to check for convergence of a series.
- Identify an alternating series.
- Explain whether an alternating series converges.
- Determine the error bound of a convergent alternating series.
- Determine whether a series converges conditionally or absolutely.
- Explain what a power series is.
- Determine the radius and interval of convergence of a power series.
- Explain what a Maclaurin Series is.
- Determine the Maclaurin Series of a given function.
- Know the Maclaurin Series for specific basic functions.
- Create new Maclaurin Series by manipulating known Maclaurin Series.
- Determine the Taylor Series for a given function.
- Apply the Lagrange Error Bound Formula.

Objectives: Students will be able to

- Identify given sequences as arithmetic or geometric. (DOK Level One)
- Apply basic properties of sequences. (DOK Level Three)
- Evaluate a series using its definition as a limit of the sequence of partial sums. (DOK Level Three)
- Identify and evaluate a geometric series. (DOK Level Two)
- Use geometric series in a variety of applied contexts. (DOK Level Three)
- Represent decimal expansions using series. (DOK Level Two)
- Evaluate a telescoping series. (DOK Level Three)
- Use graphing calculators to explore convergence or divergence of sequences and series. (DOK Level Three)
- Determine that a given series diverges using the Divergence Test. (DOK Level Two)
- Recognize the harmonic series and know that it diverges. (DOK Level One)
- Use the Comparison Test to determine whether a series converges. (DOK Level Three)
- Use the Limit Comparison Test to determine whether a series converges. (DOK Level Three)
- Write terms of series as areas of rectangles and recognize their relationship to improper integrals. (DOK Level Three)
- Use the integral test to determine whether a series converges. (DOK Level Three)
- Know that the p-series converges for $p > 1$. (DOK Level One)

- Determine whether a given p-series converges. (DOK Level Two)
- Use the Ratio Test to determine whether a series converges. (DOK Level Three)
- Determine whether an alternating series converges or diverges. (DOK Level Three)
- Find and explain the error bound of alternating series. (DOK Level Four)
- Determine and explain whether a series converges absolutely. (DOK Level Four)
- Form the power series of a function. (DOK Level Three)
- Determine the radius and interval of convergence of a power series. (DOK Level Three)
- Manipulate power series of known functions to determine power series of new functions, using methods of substitution, differentiation, antidifferentiation. (DOK Level Four)
- Recognize, write, and explain a Maclaurin series and the general Taylor series centered at $x = a$. (DOK Level Three)
- Approximate functions using a related Taylor polynomial. (DOK Level Three)
- Know the Maclaurin series for the functions $e^x, \sin x, \cos x,$ and $\frac{1}{1-x}$.(DOK Level One)
- Determine the Lagrange error bound for Taylor polynomials. (DOK Level Three)

Core Activities and Corresponding Instructional Methods:

- Sequences and series
    - Define a sequence as a function whose domain is the set of natural numbers
    - Explain the convergence/divergence of sequences using limits and techniques for evaluating limits
    - Define a series as the sum of the terms of a sequence
    - Student investigation and discovery of the convergence/divergence of series
    - Develop the idea of convergent series by analyzing the limit of the partial sums of the series
    - Evaluate telescoping series by analyzing the limit of the partial sums of the series
    - Define geometric series and derive the formula for evaluating a convergent geometric series
    - Guided practice with applying geometric series to real world problems

- Present the harmonic series and prove that it diverges
- Student investigation and discovery of the convergence/divergence of series by comparing them to known convergent/divergent series
- Explain the Comparison Test and the Limit Comparison Test
- Direct instruction on the use of Riemann Sums to associate infinite series with improper integrals
- Guided practice with applying the Integral Test to infinite series
- Direct instruction on the application of the Integral Test to determine when a p-series converges
- Student investigation and discovery of alternating series and estimating the error in using a partial sum to estimate the actual value
- Direct instruction on the difference between conditional and absolute convergence
- Guided and independent practice with determining whether given series converge using all the of the presented methods
- Student group practice with determining whether given series converge using all the of the presented methods and peer assessment of student progress.

- Power, Maclaurin, and Taylor Series
  - Direct instruction on power series as a representation of a function
  - Direct instruction on applying the Ratio Test to determine the interval of convergence of a power series
  - Direct instruction of applying various techniques to check the endpoints of an interval of convergence for a power series to determine whether the series converges at these endpoints
  - Guided and independent practice with determining the interval of convergence for a power series
  - Present a Maclaurin Polynomial as an approximating function that matches derivatives with the original function when $x = 0$

- Graph the first few terms of the Maclaurin Series for $\sin x$ on a graphing calculator to demonstrate how each successive term of the series improves the accuracy of the approximating polynomial near $x = 0$

- Derive the Maclaurin Series for $e^x, \sin x, \cos x,$ and $\frac{1}{1-x}$.

- Manipulate the Maclaurin Series for $e^x, \sin x, \cos x,$ and $\frac{1}{1-x}$ to determine Maclaurin Series for other functions.

- Guided and independent practice with developing and manipulating Maclaurin Series

- Derive the Taylor Series for $\sin x$ centered about $x = \pi$ and graph the first few terms on a graphing calculator to demonstrate how each successive term of the series improves the accuracy of the approximating polynomial near $x = \pi$

- Guided practice with deriving the Taylor Series for a given function

- Direct instruction on how to use the Lagrange Error Bound Formula to estimate the error in using a given Taylor Series to approximate a function

- Guided and independent practice with applying Maclaurin and Taylor Series to previous AP Exam problems

- Student group practice with applying Maclaurin and Taylor Series to previous AP Exam problems and peer assessment of student progress.

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework
- Quizzes

Summative: Common Assessment for Unit 6

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 434-443, p. 472-529)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, (p. 434-443, p. 472-529)
- TI-84 graphing calculator
- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

Unit 7: Parametric and Polar Equations

Time Range in Days: 10 days

Overview: This unit demonstrates and analyzes curves that are presented in parametric or polar form.

Focus Questions:
- What are parametric equations?
- How are planar curves determined by their parametric equations?
- How can calculus be applied to curves that are defined by parametric equations?
- How can the graphing calculator be utilized to graph parametric equations?
- What are polar coordinates?
- When and why are polar equations useful?
- How are planar curves determined by polar equations?
- How can calculus be applied to curves that are defined by polar equations?
- How can the graphing calculator be utilized to graph polar equations?

Goals: Students will be able to
- Define parametric equations
- Analyze planar curves that are defined by parametric equations using calculus techniques
- Graph curves that are determined by parametric equations using a graphing calculator
- Plot points using polar coordinates
- Graph basic polar equations
- Convert equations from polar form to the rectangular coordinate system
- Use a graphing calculator to plot polar curves
- Analyze polar equations using calculus techniques

Objectives: Students will be able to
- Analyze planar curves given in parametric form. (DOK Level Three)
- Find derivatives of parametric and vector functions. (DOK Level Two)
- Determine the equation of the tangent line to a parametric curve. (DOK Level Three)
- Find the second derivative of a curve defined by parametric equations and use it to interpret the concavity of the curve. (DOK Level Four)
- Find the length of a parametric curve. (DOK Level Three)
- Apply properties of vector operations. (DOK Level Three)
- Analyze planar curves given in vector form. (DOK Level Three)

- Determine and explain the velocity, acceleration, and speed of parametric curve given its position. (DOK Level Three)
- Evaluate and explain the displacement and distance traveled for a parametric curve on a given interval. (DOK Level Three)
- Analyze planar curves given in polar form. (DOK Level Four)
- Find derivatives of polar functions. (DOK Level Two)
- Determine the slope of a polar curve. (DOK Level Three)
- Calculate the area of a region bounded by polar curves. (DOK Level Three)
- Use a graphing calculator to plot curves given in polar or parametric form. (DOK Level Three)

Core Activities and Corresponding Instructional Methods:

- Parametric equations
    - Introduce parametric equations by comparing to the toy "Etch A Sketch"
    - Present examples of parametric equations and plot some sample points to investigate the shape of the curve
    - Apply derivatives ($\frac{dx}{dt}, \frac{dy}{dt}, \frac{dy}{dx},$) to analyze the shapes of curves defined parametrically and equations of their tangent lines
    - Student investigation and discovery of graphs of parametric equations using a graphing calculator
    - Explain the motion of a particle moving along a parametrically defined curve using derivatives to determine the velocity, speed, and acceleration
    - Derive the formula for the length of a curve defined parametrically
    - Guided and independent practice with the analysis of parametric equations
    - Student group practice with applying parametric equations to previous AP Exam problems and peer assessment of student progress.

- Polar equations
    - Introduce parametric equations by comparing to the toy "Spirograph"
    - Present the polar coordinate system and the equations that relate polar coordinates to rectangular coordinates
    - Student investigation and discovery of graphs of basic polar equations

- Apply derivatives ($\frac{dx}{d\theta}, \frac{dy}{d\theta}, \frac{dy}{dx},$) to analyze the shapes of curves defined using polar equations
- Student investigation and discovery of graphs of polar equations using a graphing calculator
- Derive the formula for the area of a region enclosed by curves defined using polar equations
- Guided and independent practice with the analysis of polar equations
- Student group practice with applying polar equations to previous AP Exam problems and peer assessment of student progress.

Assessments:

Diagnostic: (Honors) Pre Calculus Final Exam, Summer Work Test

Formative:

- Teacher observation and questioning
- Homework preparation
- Graded homework
- Quizzes

Summative: Common Assessment for Unit 7

Extensions:

- Teacher designed worksheets
- Textbook applications and extensions (p. 531-559)

Correctives:

- Teacher developed remediation practice worksheets
- More extensive direct instruction

Materials and Resources:

- Finney, p. 531-559
- TI-84 graphing calculator

- Previous Advanced Placement exams
- TI SmartView software
- Teacher developed worksheets
- Smart Notebook Gallery Essentials
- Geometer's Sketchpad

Unit 8: Advanced Placement Exam Preparation

Time Range in Days: 15 days

Overview: Reteach and review topics, concepts, and typical problems that will be emphasized on the Advanced Placement Exam.

Core Activities:

- Reteach topics that emphasize the key ideas of calculus: limits/continuity, rates of change (derivatives) and their applications, integrals and their applications, series, parametric equations, and polar equations.
- Distinguish between what the College Board considers AB topics and what are considered C topics
- Review essential features on the graphing calculator and how to use them in solving and analyzing calculus problems.
- Provide extra practice problems in preparation for the AP Exam.
- Solve and analyze problems from previous AP Exams.
- Assign and discuss sample AP Exam problems provided by the College Board or from other sources.
- Group practice with sample AP Exam problems and peer assessment of student progress.
- Assess students' exam readiness using review quizzes, graded homework assignments, and various multiple-choice and free response questions, as deemed necessary.
- Discuss test-taking strategies.

Unit 9: Enrichment Calculus Concepts

Time Range in Days: 30 days

Overview: Further calculus topics selected by the teacher will be taught after the Advanced Placement Exam is administered in May. These topics may include applications of differential equations (predator-prey model), applications of Taylor Series, paradoxes involving infinite series, and plotting of 3-dimensional graphs using either a graphing calculator with such capabilities (such as TI-89) or “Derive” software.

Core Activities:

- Use concepts and techniques from throughout the course to solve applied problems or to study advanced calculus topics that are not required for the Advanced Placement Calculus (BC) Exam.

Advanced Placement Calculus (BC) Topic Outline
Established by the College Board

Analysis of graphs

Students will connect geometric and analytic information and use calculus both to predict and to explain the observed local and global behavior of a function.

Limits of functions

Students will have an intuitive understanding of the limiting process

Students will calculate limits (including one-sided limits) using algebra.

Students will estimate limits from graphs or tables of data.

Asymptotic behavior

Students will understand asymptotes in terms of graphical behavior

Students will describe asymptotic behavior in terms of limits involving infinity

Students will compare relative magnitudes of function and their rate of change. (For example, contrasting exponential growth, polynomial growth, and logarithmic growth.)

Continuity as a property of functions

Students will have an intuitive understanding of continuity. (Close values of the domain lead to close values of the range.)

Students will understand continuity in terms of limits

Students will have a geometric understanding of graphs of continuous functions.

Students will apply the Intermediate Value Theorem and the Extreme Value Theorem.

Concept of a derivative

Students will present derivatives graphically, numerically, and analytically.

Students will interpret the derivative as an instantaneous rate of change.

Students will apply the definition of a derivative as the limit of the difference quotient.

Students will recognize the relationship between differentiability and continuity.

Parametric, polar, and vector functions

Students will analyze planar curves including those given in parametric form, polar form, and vector form.

Derivative at a point

Students will find the slope of a curve at a point.
Students will determine whether a curve has points at which there is a vertical tangent or no tangent.
Students will use a tangent line to approximate values of a function.
Students will calculate instantaneous rate of change by using the limit of average rate of change.
Students will approximate rate of change of a function from graphs and table of values.

Derivative as a function

Students will distinguish between the characteristics of f and f'.
Students will understand the relationship between the increasing and decreasing behavior of f and the sign of f'.
Students will apply the Mean Value Theorem and its geometric consequences.
Students will solve equations involving derivatives.
Students will translate verbal descriptions into equations involving derivatives and vice versa.
Students will use L'Hopital's Rule to determine limits.

Second derivatives

Students will distinguish between the corresponding characteristics of f, f', and f''.
Students will understand the relationship between the concavity of f and the sign of f''.
Students will recognize inflection points as places where concavity changes.

Applications of derivatives

Students will analyze curves, including the idea of monotonicity and concavity.
Students will analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration vectors.
Students will optimize functions, finding both absolute (global) and relative (local) extrema.
Students will model rates of change, including related rates problems.
Students will use implicit differentiation to find the derivative of an inverse function.
Students will interpret the derivative as a rate of change in varied applied contexts.
Students will use derivatives to solve linear motion problems involving position, velocity, speed, and acceleration.

Computation of derivatives

Students will find derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.
Students will find derivatives of functions using basic rules for sums, products, and quotients.
Students will find derivatives of functions using the chain rule and implicit differentiation.
Students will find derivatives of parametric, polar, and vector functions.

Interpretations and properties of definite integrals

Students will estimate definite integrals by computing Riemann Sums using left, right, and midpoint evaluation points.
Students will evaluate a definite integral by calculating the limit of corresponding Riemann Sums over equal subdivisions.
Students will interpret a definite integral of a rate of change of a quantity as the change of the quantity over the interval
Students will apply basic properties, such as additivity and linearity, to definite integrals.

Applications of integrals

Students will apply the concept of using an integral of a rate of change to find accumulated change.
Students will calculate the area of a given region.
Students will calculate the volume of a solid with known cross-sections.
Students will calculate the average value of a function.
Students will calculate the distance traveled by a particle along a line.
Students will find the area of a region bounded by polar curves.
Students will find the length of planar curves, including those given in parametric form.

Fundamental Theorem of Calculus

Students will use the Fundamental Theorem to evaluate definite integrals.
Students will use the Fundamental Theorem to represent a particular antiderivative.
Students will analyze both analytically and graphically functions defined as definite integrals.
Students will evaluate improper integrals.

Techniques of antidifferentiation

Students will find antiderivatives of functions directly from derivatives of basic functions.
Students will find antiderivatives of functions by substitution of variables, including change of limits for definite integrals.

Applications of antidifferentiation

Students will find specific antiderivatives using initial conditions, including applications to motion along a line.
Students will solve separable differential equations and use them to model solutions.
Students will interpret differential equations geometrically by using slope fields.
Students will sketch solution curves for differential equations using slope fields.
Students will numerically approximate solutions to differential equations using Euler's method.
Students will solve exponential growth and decay problems.
Students will solve logistic differential equations and use them in modeling.

Numerical approximations to definite integrals

Students will use trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.

Concept of series

Students will apply the definitions of series and convergence.
Students will use technology to explore convergence or divergence.

Series of constants

Students will use series to represent decimal expansion.
Students will evaluate geometric series with applications.
Students will identify the harmonic series and know that it diverges.
Students will evaluate partial sums of alternating series and apply the error bound.
Students will represent terms of a series as areas of rectangles and recognize their relationship to improper integrals, including the integral test and its use in testing the convergence of *p*-series.
Students will apply the ratio test for convergence and divergence.
Students will apply the comparison test for convergence and divergence.
Students will apply the limit comparison test for convergence and divergence.
Students will distinguish between conditional and absolute convergence.

Taylor Series

Students will graphically use a Taylor polynomial to approximate a function.
Students will formulate Maclaurin series and the general Taylor series about $x = a$.
Students will identify the Maclaurin series for $e^x, \sin x, \cos x, and\ \frac{1}{1-x}$.
Students will manipulate Taylor series and use shortcuts to compute Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.
Students will use functions defined by power series.
Students will determine the radius and interval of convergence of power series.
Students will determine the Lagrange error bound for Taylor polynomials.

Graphing Calculators

Students will plot the graph of a function within an arbitrary window.
Students will find the zeros of a function (solve equations numerically).
Students will numerically calculate the derivative of a function.
Students will numerically calculate the value of a definite integral.